Limit Cycle Prediction in Multivariable Nonlinear Systems Using Genetic Algorithms

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Abstract. This paper presents an intelligent method based on multiuobjective genetic algorithm (MOGA) for prediction of limit cycle in multivariable nonlinear systems. First we address how such the systems may be investigated using the Single Sinusoidal Input Describing Function (SIDF) philosophy. The extension of the SIDF to multi loop nonlinear systems is presented. For the class of separable nonlinear element of any general form, the harmonic balance equations are derived. A numerical search based on multiobjective genetic algorithm is addressed for the direct solution of the harmonic balance system matrix equation. The MOGA is employed to solve the multiobjective formulation and obtain the quantitative values for amplitude, frequency and phase difference of possible limit cycle operation. The search space of MOGA is the space of the possible limit cycle parameters, such as amplitudes, frequency and phase difference between the interacting loops. Finally computer simulation is performed to show how the analysis given in the paper is used to predict the existence of the limit cycle of the multivariable nonlinear systems.

1 Introduction

The frequency response method is a powerful tool for the analysis and design of linear control systems. It is based on describing a linear system by a complex-valued function instead of differential equation. However, frequency domain analysis can not be directly applied to nonlinear systems because frequency response functions can not be defined for nonlinear systems. The Describing Function (DF) method is an extended version of the frequency response method, which can be used to approximately analyze and predict nonlinear behavior[3,2]. The main use of describing function method is for the prediction of limit cycles (oscillations) in nonlinear systems [1]. A limit cycle is the phenomenon that can be observed in systems composed of nonlinear elements. The phenomenon is of fundamental importance in nonlinear systems and, as far as the design of a nonlinear system is concerned, it should be considered along with the stability analysis [6]. The applicability of DF to limit cycle analysis is due to the fact

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that the form of the signals in the limit-cycling system is usually approximately sinusoidal. In fact, we assume that the linear part of the system has low-pass properties, which can attenuate the harmonics of the nonlinear system.
In this paper the describing function method is extended to multi-loop systems and the MOGA formulation is designed to numerically search for limit cycles. The computer simulations are performed to show how the proposed method is used to predict the existence of the limit cycle of the nonlinear multivariable systems.
The configuration of this paper is as follow: Section 2 offers a brief summary of the sinusoidal input describing function theory, because it will be used in the sequel. In section 3 the definition of limit cycle as a characteristic of nonlinear systems is presented. The extension of the philosophy of the harmonic balance equation to multivariable nonlinear systems will discussed in section 4. Several approaches have been formulated to analyze and predict the limit cycle oscillation in nonlinear systems, in section 5 a brief review of multiobjective genetic algorithm that we will use for predicting of amplitude, frequency and phase of limit cycle of nonlinear MIMO systems, is presented. Simulation results and some concluding remarks is discussed in next sections.

2 Sinusoidal Input Describing Function

The Sinusoidal Input Describing Function (SIDF) approach generally can be used to study periodic phenomena. It is applied for two primary purposes: limit cycle analysis and characterizing the input/output behavior of a nonlinear plant in the frequency domain. In short a SIDF describes the amplitude and phase of the first harmonic of the periodic output signal of the nonlinear system with respect to the sinusoidal input signal [4]. Due to the characteristics of a nonlinear system, the SIDF will be dependent on both the amplitude and the frequency of the input signal. The SIDF can easily be measured with a Dynamic Signal Analyzer, which generates a sinusoidal source signal with a presanctified amplitude and frequency. The corresponding amplitude and phase of the output signal are computed online, resulting in the SIDF for the specified input amplitude and frequency. In order to develop the basic version of the describing function method, the system has to satisfy the following conditions [5]:

1. There is only a single nonlinear component and the system can be rearranged into the form shown in Figure 1.
2. The nonlinear component is time-invariant.
3. Corresponding to a sinusoidal input, only the fundamental component of the output is considered.
4. The nonlinearity is odd.

If the input of the nonlinear system is a sine wave $e(t) = A\sin(\omega t)$, then the output is periodic and can be expressed as:

$$u(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (1)$$

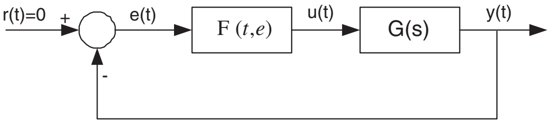

Fig. 1. Nonlinear System for Describing Function Analysis

where the coefficients a_i's and b_i's are generally functions of A and ω determined by:

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} u(\theta) d\theta$$
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} u(\theta) \cos(n\theta) d\theta$$
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} u(\theta) \sin(n\theta) d\theta \tag{2}$$

where $\theta = \omega t$. Because of our assumptions, $a_0 = 0$, $n = 1$, and

$$u(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t) = M(A, \omega) \sin(\omega t + \varphi(A, \omega)) \tag{3}$$

where

$$M(A, \omega) = \sqrt{a_1^2 + b_1^2} \quad \text{and} \quad \varphi(A, \omega) = \arctan(\frac{a_1}{b_1}) \tag{4}$$

From the above equations it can be seen that the fundamental component of the output corresponding to a sinusoidal input is a sinusoid of the same frequency and can be written as:

$$M(A, \omega) e^{j(\omega t + \varphi(A, \omega))} = (b_1 + j a_1) e^{j(\omega t)} \tag{5}$$

The describing function of the nonlinear element is defined as the complex ratio of the fundamental component of the nonlinear element by the input sinusoid, i.e.

$$N(A, \omega) = \frac{M(A, \omega) e^{j(\omega t + \varphi(A, \omega))}}{A \sin(\omega t)} = \frac{1}{A}(b_1 + j a_1) \tag{6}$$

By replacing the nonlinear element, $F(e)$,in figure 1, with its describing function, $N(A, \omega)$, the nonlinear element can be treated as if it where a linear element with a frequency response function. As can be seen from equation (6), Generally, the describing function depends on the frequency and amplitude of the input signal.

3 Limit Cycle Prediction for Nonlinear Systems

A limit cycle is a periodic signal, $x_{LC}(t + T) = x_{LC}(t)$ for all t and some T (the period) such that perturbed solutions either approach x_{LC} (a stable limit cycle) or diverge from it (an unstable one). An approach to limit cycle analysis that has gained widespread acceptance is the frequency-domain / SIDF method. In Figure 1 if we replace $F(e)$ with $N(A,\omega)$ and assume that a self-sustained oscillation of amplitude A and frequency ω exists in the system then for $r = 0$, $y \neq 0$, we have:

$$N(A,\omega)G(j\omega) + 1 = 0 \quad \text{or} \quad N(A,\omega)G(j\omega) = -1 \tag{7}$$

This equation, called the *harmonic balance equation* [7]. If any limit cycles exist in our system, and the four assumptions (mentioned in section 2) are satisfied, then the amplitude and frequency of the limit cycles can be predicted by solving the *harmonic balance equation.* If there are no solutions to the harmonic balance equation then the system will have no limit cycles (under the above assumptions). It is generally very difficult to solve this equation by analytical methods.

3.1 Limit Cycle Prediction for Nonlinear Multivariable Systems

Provided that certain limitation are placed on the form of the linear system elements, the extension of the philosophy of harmonic linearization to multi-variable systems is conceptually straightforward and has been suggested by several authors [1,2]. The equation governing limit cycle operation in the autonomous multi-variable nonlinear feedback system of figure 1 can be expressed as:

$$det\Big(\tilde{N}(A,\omega)G(j\omega) + I\Big) = 0 \tag{8}$$

Where $\tilde{N}(A,\omega)$ is a matrix of single sinusoidal input describing functions corresponding to the nonlinear elements of $N(A,\omega)$. Thus for no limit-cycle to exist no eigenvalue of $\tilde{N}(A,\omega)G(j\omega)$ can equal $(-1, j0)$. Consider the 2×2 system shown in Figure 2, the set of equation governing this model is given by equation (9).

$$\begin{aligned}(1 + n_{11}g_{11})A_1 + (n_{12}g_{12})A_2e^{j\varphi} = 0\\(n_{21}g_{21})A_1 + (1 + n_{22}g_{22})A_2e^{j\varphi} = 0\end{aligned} \tag{9}$$

The solution of this equation is sought for specific values of A_1, A_2, ω and φ, where A_1 and A_2 are amplitudes of limit cycles in loop 1 and 2 respectively, ω is frequency of oscillation for both loops and φ is phase-shift between the loops. In this case there are two equations with four unknown variables. Equation (9) can be solved by searching the space of A_1, A_2, ω and φ. As we mentioned above, solving the harmonic balance equation is not trivial; for higher order systems the analytical solution is very complex.

In this paper the Multiobjective Genetic Algorithm (MOGA) is used to search over the existence of any possible limit cycle operation in nonlinear systems and subsequently over controller structures as well as over the controller parameters.

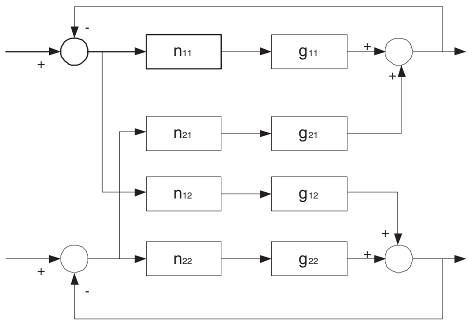

Fig. 2. A two-inputs two-outputs nonlinear system

4 Multiobjective Evolutionary Algorithms

The Genetic Algorithms (GAs) are the stochastic global search method that mimic the metaphor of natural biological evolution. These algorithms maintain and manipulate a population of solutions and implement the principle of survival of the fittest in their search to produce better and better approximations to a solution. This provides an implicit as well as explicit parallelism that allows for the exploitation of several promising areas of the solution space at the same time. The implicit parallelism is due to the schema theory developed by Holland, while the explicit parallelism arises from the manipulation of a population of points [8]. The implementation of GA involves some preparatory stages. Having decoded the chromosome representation into the decision variable domain, it is possible to assess the performance, or fitness, of individual members of a population. This is done through an objective function that characterizes an individual's performance in the problem domain. During the reproduction phase, each individual is assigned a fitness value derived from its raw performance measure given by objective function. Once the individuals have been assigned a fitness value, they can be chosen from population, with a probability according to their relative fitness, and recombined to produce the next generation. Genetic operators manipulate the genes. The recombination operator is used to exchange genetic information between pairs of individuals. The crossover operation is applied with a probability p_x when the pairs are chosen for breeding. Mutation causes the individual genetic representation to be changed according to some probabilistic rule. Mutation is generally considered to be a background operator that ensures that the probability of searching a particular subspace of the problem space is never zero. This has the effect of tending to inhibit the possibility of converging to a local optimum.

Multiobjective Evolutionary Algorithms (MOEA) are based on multi-objective Genetic Algorithms (MOGA). The MOEA begins with a population of possible solutions, called strings. Each string is fed into a model as the candidate solution, in this case these strings are the parameters of the configured controller model.

This model is usually a computer program representation of the solution to the problem. Multi-objective simply means that there is more than one objective involved [9]. For each string, each objective represents a separate cost. The manner in which a string is deemed superior or inferior to other strings is carried out by a selection mechanism. The selected strings undergo the evolutionary process where the traits of the selected strings (which may or may not be good) are selected and combined to form new strings for the next generation. In theory, with each generation, the strings in the population should return better and better cost functions by obtaining strings nearer to the optimal solutions. In practice, often there are limits to the values of cost functions that can be achieved. This depends on the objective functions and the constraints imposed on the model parameters.

5 Simulation Results

The following simulation results illustrate the capabilities of proposed MOGA for limit cycle prediction. In these simulations we choose two 2×2 nonlinear systems. In both systems the numerical solution of equation (9) is formulated as two objective problems. One is absolute value of the first equation and the second objective is simply the absolute of the second equation in (9), as shown in equation (10). The MOGA searches in the space of A_1, A_2, ω and φ to find a set of such parameters that minimize both objectives. Due to the inherent approximation in using SIDF and also the nature of multi-objective formulation, it may not be possible to reach the exact minimum ,which is zero, as required by the equation set (10). Therefore MOGA may converge to a set of Parteo Optimal solution, so in the two nonlinear systems four of the best obtained results were selected as the final results.

$$
\begin{aligned}
Objectiv_1 &= |(1 + n_{11}g_{11})A_1 + (n_{12}g_{12}A_2)e^{j\varphi}| \\
Objectiv_2 &= |(n_{21}g_{21})A_1 + (1 + n_{22}g_{22})A_2e^{j\varphi}|
\end{aligned}
\tag{10}
$$

5.1 Simulation Results for the First System

The block diagram of the first system has shown in figure 3. As can be seen, this diagram has two linear and nonlinear parts. The elements of the nonlinear part are similar and consist of four ideal relays but the elements of the linear part consists of four linear transfer functions. Parameters setting of MOGA for this system has given in Table 1. The obtained results for prediction of limit cycle

Table 1. MOGA parameters for the first nonlinear system

No. of generation	Population size	Mutation	Crossover
74	36	0.01	0.94

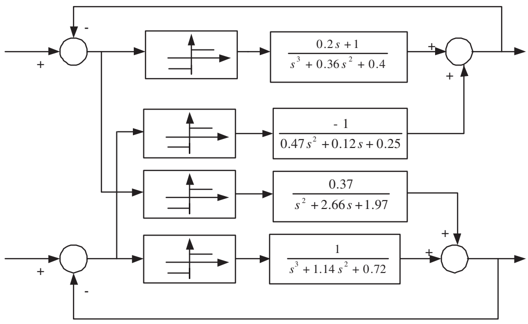

Fig. 3. The block diagram of the first system

of this system has presented in Table 2. With inspection of table 2, we can see that the obtained results of proposed approach is too close to analytical method, which describes the salification of our method.

5.2 Simulation Results for the Second System

The block diagram of the second nonlinear system has shown in figure 4. As can be seen, its configuration is similar to the first system, but the nonlinear matrix ,in additional of ideal relay, consists of different nonlinear behavior such as saturation and dead zone. The parameters setting of MOGA for this system

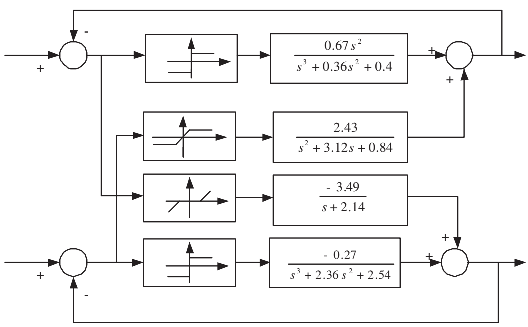

Fig. 4. The block diagram of the second system

are those parameters given in table 1. The obtained results for prediction of limit cycle of this system has presented in Table 3. With inspection of table 3, we can see that the obtained results of proposed approach is too close to analytical method. All programs of the MOGA method have been written in MATLAB/SIMULINK software and c++ language. These programs have been

Table 2. Obtained results of limit cycle prediction with MOGA for the first system

Best obtained results	ω	A_1	A_2	φ	$Objective_1$	$Objective_2$
1	0.37654	1.45627	0.86423	2.35647	0.01162	0.02317
2	0.37622	1.45638	0.86475	2.35609	0.01768	0.02406
3	0.37697	1.45704	0.86429	2.35698	0.01967	0.02385
4	0.37604	1.45677	0.86448	2.35637	0.01145	0.01249
Analytical method	0.37596	1.45668	0.86437	2.35643	–	–

Table 3. Obtained results of limit cycle prediction with MOGA for the second system

Best obtained results	ω	A_1	A_2	φ	$Objective_1$	$Objective_2$
1	0.64725	0.87654	1.28343	3.48752	0.01422	0.01963
2	0.64711	0.87637	1.28387	3.48727	0.01271	0.01537
3	0.64787	0.87672	1.28376	3.48794	0.03624	0.02415
4	0.64762	0.87616	1.28307	3.48765	0.03245	0.02714
Analytical method	0.64716	0.87652	1.28367	3.48713	–	–

executed on a Pentium IV personal computer. On this computer, the response time of the proposed method for all testing cases, was less than 83 second, which makes feasible the application of the proposed approach for limit cycle prediction in reasonable and acceptable computation time.

6 Conclusion

In this paper an intelligent method of predicting limit cycle amplitude, frequency, and phase for nonlinear multivariable systems was presented. this method was based on multiobjective genetic algorithm, which was capable of predicting specified modes of theoretical limit cycle operation. An advantage of this method ,as we showed, was that MOGA can be directed to search for all possible solutions including sub-harmonic components that are ignored in the derivation of the SIDF. Furthermore the proposed method was capable of quantifying the magnitude, frequency and the phase of the limit cycles as well as the loop interaction effects in the frequency domain which proves useful in any subsequent controller design. The effectiveness of the proposed method was demonstrated trough examples. Obtained results showed that MOGA can be used for predicting of limit cycle in MIMO systems with high nonlinearity in a reasonable and acceptable computation time.

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